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# FLUIDIZATION IN THE PRESENCE OF AN OBSTACLE

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The problem of transition of a friable medium layer into a suspended state in the presence in it of an internal obstacle is considered. Problems of this kind are frequently encountered in practice in connection with heterogeneous catalytic reactions in reactors with suspended catalyst layer, heat exchangers, surface coating operations which involve the immersion of articles in a fluidized layer, etc. Critical regions of the onset of fluidized state and the critical velocity of the stream are determined by the general method described in [1, 2]. Results of experiments on the fluidization of a layer with a cylindrical obstacle are presented. Comparison of theoretical and experimental data shows a good agreement.

**1.** The problem considered here is a particular case of the general problem of fluidization onset [1, 2]. The latter reduces to the problem of the limit equilibrium of a body whose resistance to tensile stress does not exceed a certain limit $\sigma_s$ which is constant for a particular friable medium and, generally, nonzero.

It was shown earlier [1, 2] that in the case of the plane problem the lines of principal stress along which normal stress components at small areas tangent to these attain their maximum $\sigma_s$, while all shear stresses are zero, coincide with the integral curves $x = x_1(\xi, \eta)$, $y = x_2(\xi, \eta)$ of equation

$$adx_2 = bdx_1 \tag{1.1}$$

(Condition $\eta = \text{const}$ separates one line of the set, $x$ and $y$ are Cartesian coordinates, and the $\xi$-coordinate is measured along the line $\eta = \text{const}$). Here $a$ and $b$ are components of the body force vector acting on the friable body in directions $x_1$ and $x_2$ and taken with the opposite sign. The body force is

$$Q = d^* F - \text{grad } p \tag{1.2}$$

where $d^*$ is the density of the two-phase system, $F$ is the external mass force, and $p$ is the pressure in the fluid phase.

In the axisymmetric problem [2] Eq. (1.1) similarly determines the surfaces of principal stresses $r = x_1(\xi, \eta)$ and $z = x_2(\xi, \eta)$ in the cylindrical system of coordinates $r, \varphi, z$. The only unknown stress $\sigma_\xi$ along the line (surface) of principal stresses is of the form

$$\sigma_\xi = \frac{1}{r^\gamma H_2(\xi, \eta)} \left[ A(\eta) + \int_{\xi_0}^{\xi} Q_\xi r^\gamma H_2(\xi, \eta) \, d\xi \right] \tag{1.3}$$

$$H_2^2(\xi, \eta) = (\partial x_1 / \partial \eta)^2 + (\partial x_2 / \partial \eta)^2$$

where the arc length $\eta$ is taken as the coordinate $\xi$, $\varphi = \text{const}$, $\gamma = 0$ and 1 for the plane and the axisymmetric problems, respectively, and function $A(\eta)$ is determined by boundary conditions. The critical point of fluidization onset and the critical values of stream parameters are determined by the condition $\sigma_\xi = \sigma_s$ inside the layer.

Let us now make the following fairly general assumptions:

(1) the friable medium is homogeneous, (2) the fluid is incompressible, and (3) the field of external mass forces is solenoidal. As will be shown below, it is possible to write out on these assumptions the integral of Eq. (1.1), eliminate in (1.3) the quadrature, and obtain the condition for determining critical parameters in a simple form convenient for practical application.

By virtue of the second and third assumptions it is possible to express the components of the filtration rate $v$ and of the external mass force vector $F$ in terms of functions $\psi^{(1)}(x_1, x_2)$ and $\psi^{(2)}(x_1, x_2)$, respectively, as

$$v_1 = r^{-\gamma} \partial \psi^{(1)} / \partial x_2, \quad v_2 = -r^{-\gamma} \partial \psi^{(1)} / \partial x_1 \tag{1.4}$$

$$F_1 = r^{-\gamma} \partial \psi^{(2)} / \partial x_2, \quad F_2 = -r^{-\gamma} \partial \psi^{(2)} / \partial x_1$$

Using Darcy's law, the formula (1.2) for the body force, and formula (1.4) for the components of the body force taken with inverse sign, we obtain

$$a = -\frac{\mu}{k} \frac{1}{r^\gamma} \frac{\partial \psi^{(1)}}{\partial x_2} - (1 - \varepsilon)(d_2 - d_1) \frac{1}{r^\gamma} \frac{\partial \psi^{(2)}}{\partial x_2} \tag{1.5}$$

$$b = \frac{\mu}{k} \frac{1}{r^\gamma} \frac{\partial \psi^{(1)}}{\partial x_1} + (1 - \varepsilon)(d_2 - d_1) \frac{1}{r^\gamma} \frac{\partial \psi^{(2)}}{\partial x_1}$$

where $\mu$ is the dynamic viscosity coefficient of the fluid, $k$ is the permeability, $\varepsilon$ is the porosity of the friable medium, and $d_2$ and $d_1$ are the densities of the solid particles and fluid, respectively.

It follows from assumption (1) and formulas (1.5) that (1.1) is an equation in total differentials whose general integral is of the form

$$\psi^{(1)}(x_1, x_2) + E \psi^{(2)}(x_1, x_2) = \eta \tag{1.6}$$

$$E = \frac{k}{\mu} (1 - \varepsilon)(d_2 - d_1)$$

Equation (1.6) defines the set of the sought lines (surfaces) of principal stresses.

Note that function $\psi^{(1)}(x_1, x_2)$ is determined by the solution of the filtration problem which reduces to the determination of a harmonic function in the region occupied by the friable medium for specified conditions at the boundaries. The determination of function $\psi^{(2)}(x_1, x_2)$ in the cases which are important in practical applications (fluidization in a gravitational or centrifugal field, etc.) do not usually present difficulties.

Using the general integral (1.6), for the Lamé coefficient $H_2$ we obtain

$$H_2 = \left[\left(\frac{\partial\psi^{(1)}}{\partial x_1} + E\frac{\partial\psi^{(2)}}{\partial x_1}\right)^2 + \left(\frac{\partial\psi^{(1)}}{\partial x_2} + E\frac{\partial\psi^{(2)}}{\partial x_2}\right)^2\right]^{-1/2} = \frac{\mu}{k}\frac{r^{-\gamma}}{\sqrt{a^2+b^2}}$$

It becomes now possible to write formula (1.3) for stresses $\sigma_\xi$ in the simple form

$$\sigma_\xi = [A'(\eta) + \xi - \xi_0]\, Q_\xi \qquad (1.7)$$

where

$$A'(\eta) = k\,\mu^{-1} A(\eta)\,\mathrm{sgn}\, Q_\xi$$

Let $\xi_0$ be the length of arc of the characteristic up to the point lying on the free surface. Using the boundary condition $\sigma_\xi = 0$ at the free surface, for the critical condition of fluidization onset we then obtain

$$Q_\xi^* = \frac{\sigma_s}{\xi^* - \xi_0} \qquad (1.8)$$

The critical condition thus implies that the product of the body force along the characteristic passing through a given point inside the layer by the length of the characteristic arc from that point to the free surface must be equal to the ultimate tensile stress of the medium. Condition (1.8) determines the critical point at each characteristic. To determine the points of the beginning of fluidization, it is necessary to select points $\xi = \xi^*$, for which the fluid flow rate is minimal.

Note that in the particular case of absence of adhesion between particles, condition (1.8) reduces to the condition for the body force to vanish.

**2.** Let us consider the case of a cylindrical obstacle in the gravitational field. Let in the problem in plane formulation the obstacle have the shape of a circular cylinder whose axis is normal to the plane of flow which is uniform at infinity. The layer is assumed to be unbounded.

Solution of the related filtration problem in conditions to which Darcy's law applies, yields for the stream function the following expression:

$$\psi^{(1)} = -Ux\left(1 - \frac{a_0^2}{x^2+y^2}\right) \qquad (2.1)$$

where $U$ is the rate of filtration at infinity, $a_0$ is the cylinder radius, the origin of the Cartesian system of coordinates is located on the cylinder axis, and the direction of the axis of ordinates coincides with that of the stream of fluid at infinity and is opposite to the direction of gravity.

For function $\psi^{(2)}$ we evidently have

$$\psi^{(2)} = gx \qquad (2.2)$$

where $g$ is the acceleration of gravity.

In dimensionless coordinates normalized with respect to the cylinder radius, the set of lines of principal stresses is, in accordance with (1.6), (2.1) and (2.2), defined by the equation

$$(G-1)X+\frac{X}{X^2+Y^2}=\mathrm{H}\qquad\left(G=\frac{Eg}{U},\ \mathrm{H}=\frac{\eta}{Ua_0}\right)\tag{2.3}$$

An increase of the velocity of flow of the fluid, i.e. the decrease of parameter $G$, alters the stress field. The related evolution of principal stress lines is shown in Fig. 1 for $G = 3, 2, 1.5$ and 1 in diagrams a, b, c and d, respectively. The marked qualitative change of the field with increasing $G$ can be observed.

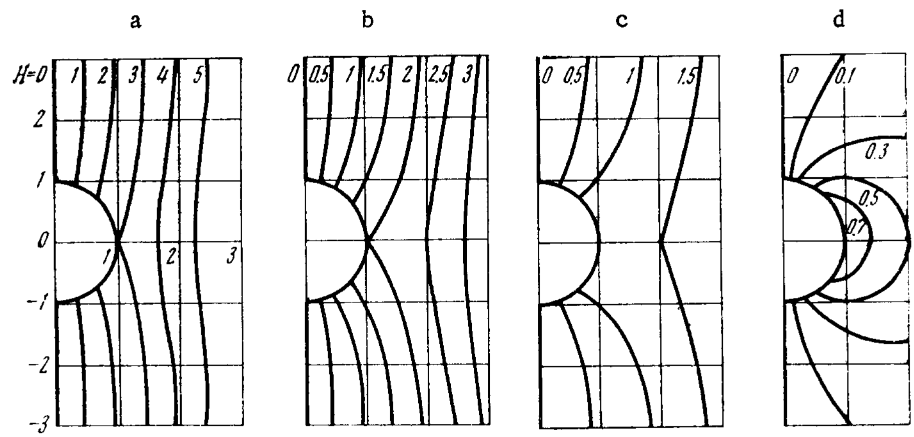


Fig. 1

In determining critical points at which fluidization begins and the related rate of fluid flow is at its minimum, we restrict our analysis for simplicity to the case of absence of adhesion between particles of the solid phase ($\sigma_s = 0$). From condition (1.9) and formulas (2.1) and (2.2) we obtain (in dimensionless coordinates)

$$\frac{2XY}{(X^2+Y^2)^2}=0,\qquad G-1+\frac{Y^2-X^2}{(X^2+Y^2)^2}=0$$

which yield two obvious systems of solutions

$$X=0,\ \ Y=\pm(1-G)^{-1/2},\quad G\leqslant 1$$
$$Y=0,\ \ X=\pm(G-1)^{-1/2},\quad G\geqslant 1$$

(in addition to the trivial solutions $X^2+Y^2\to\infty$ for $G=1$).

For the first system the maximum value of parameter $G$ is $G_{max} = 1$ (for $Y = \pm\infty$) and for the second $G_{max} = 2$ (for $X = \pm 1$). Reverting to dimensional variables, we thus conclude that in this case the fluidization onset begins at points $x = \pm a_0$, $y = 0$ at the stream critical velocity $U_* = {}^1/_2 k\,\mu^{-1}(1-\varepsilon)(d_2-d_1)\,g$, which is half of that obtaining in the absence of an obstacle. The corresponding pattern of principal stress lines is shown in Fig. 1, b.

3. To estimate the effect of the apparatus wall on the fluidized layer with an internal obstacle, we shall consider the limit case in which the cylinder is in contact with the flat wall along one of its generatrices and is normal to the stream flowing along the wall (see Fig. 2). A direct inspection shows that in this case the complex potential of

the flow is defined by
$$w = -ia_0\pi U \operatorname{ctg}\frac{a_0\pi}{z}, \qquad z = x + iy$$

hence
$$\psi^{(1)} = -a_0\pi U \sin\frac{2a_0\pi x}{r^2}\left(\operatorname{ch}\frac{2a_0\pi y}{r^2} - \cos\frac{2a_0\pi x}{r^2}\right)^{-1} \qquad (r^2 = x^2 + y^2) \qquad (3.1)$$

Consequently the set of principal stress lines, in accordance with (2.2) and (1.6), is defined by the equation (in dimensionless variables)

$$GX - \frac{1}{2}\sin\frac{X}{X^2+Y^2}\left(\operatorname{ch}\frac{Y}{X^2+Y^2} - \cos\frac{X}{X^2+Y^2}\right)^{-1} = H \qquad (3.2)$$

$$X = \frac{x}{2\pi a_0}, \quad Y = \frac{y}{2\pi a_0}, \quad H = \frac{\eta}{2\pi U a_0}, \quad G = \frac{Eg}{U}$$

Let us determine the critical points at which the fluidization begins and the critical stream parameters in the case of $\sigma_s = 0$. Using (3.1) and (3.2) we obtain by virtue of (1.9) the following conditions for the transition to fluidization:

$$(X^2+Y^2)^{-2}(\cos\Lambda_x - \operatorname{ch}\Lambda_y)^{-2}\Big[XY\cos\Lambda_x\operatorname{ch}\Lambda_y + \frac{1}{2}(X^2-Y^2)\sin\Lambda_x\operatorname{sh}\Lambda_y - XY\Big] = 0 \qquad (3.3)$$

$$G + (X^2+Y^2)^{-2}(\cos\Lambda_x - \operatorname{ch}\Lambda_y)^{-2}\times\Big[\frac{1}{2}(X^2-Y^2)\cos\Lambda_x\operatorname{ch}\Lambda_y - XY\sin\Lambda_x\operatorname{sh}\Lambda_y - \frac{1}{2}(X^2-Y^2)\Big] = 0$$

$$\Lambda_x = \frac{X}{X^2+Y^2}, \qquad \Lambda_y = \frac{Y}{X^2+Y^2}$$

where the dimensionless variables are as defined in (3.2).

Solutions of system (3.3) are of the form
$$X = 0, \quad G + {}^1/_2\, Y^{-2}(1 - \operatorname{ch} Y^{-1})^{-1} = 0$$
$$Y = 0, \quad G - {}^1/_2\, X^{-2}(1 - \cos X^{-1})^{-1} = 0$$

It follows from this that the sought maximum value of parameter $G$ is obtained at the point $X = 1/\pi$, $Y = 0$ and is $\quad G_{\max} = \pi^2/4$

Passing to dimensional variables we conclude that in this case fluidization begins at point $x = 2a_0$, $y = 0$ at a stream velocity which is 2.47 times lower than the velocity at which the beginning of fluidization occurs in the absence of an obstacle. It will be also seen, that in the proximity of the apparatus wall the critical velocity is reduced by 19% in comparison with the case of an obstacle in an unbounded layer (see (Sect. 2).

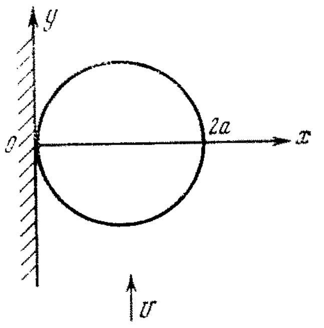

Fig. 2

Note that the above case also corresponds to the problem of an obstacle in the form of two cylinders in contact along their generatrices and normal to the stream.

4. As an example of the axisymmetric problem let us consider a spherical obstacle. In accordance with formulas (1.4) for functions $\psi^{(1)}$ and $\psi^{(2)}$ we have

$$\psi^{(1)} = -\frac{1}{2} Ur^2 \left[1 - \frac{a_0^3}{(r^2 + z^2)^{3/2}}\right] \qquad (4.1)$$

$$\psi^{(2)} = \frac{1}{2} gr^2$$

where $a_0$ is the radius of the sphere, and $r$ and $z$ are cylindrical coordinates (the origin of coordinates is at the center of the sphere, and the direction of the $z$-axis coincides with that of the stream velocity at infinity).

In accordance with (1.6) and (4.1) we obtain for the set of principal stress surfaces an equation of the form

$$(G - 1) R^2 + \frac{R^2}{(R^2 + Z^2)^{3/2}} = H \qquad (4.2)$$

$$R = \frac{r}{a_0}, \quad Z = \frac{z}{a_0}, \quad G = \frac{Eg}{U}, \quad H = \frac{2\eta}{Ua_0^2}$$

The pattern of stress field variation with increasing $G$ is here similar to that in the case of a cylinder (see Fig. 1).

To determine the critical points of minimum flow rate which corresponds to fluidization onset from (1.8) we obtain in the case of $\sigma_s = 0$ the following conditions:

$$\frac{3R^2Z}{(R^2 + Z^2)^{5/2}} = 0, \quad 2(G - 1) + \frac{2Z^2 - R^2}{(R^2 + Z^2)^{5/2}} = 0$$

As in the case of a cylinder this yields two systems of solutions

$$R = 0, \quad Z = \pm (1 - G)^{-1/3} \quad G \leqslant 1$$

$$Z = 0, \quad R = [2(G - 1)]^{-1/3}, \quad G \geqslant 1$$

(in addition to trivial solutions $R^2 + Z^2 \to \infty$ for $G = 1$).

Thus the maximum possible value of parameter $G$ is $G_{max} = 3/2$ which obtains for $Z = 0$ and $R = 1$. Reverting to dimensional variables we conclude that fluidization in the presence of a spherical obstacle takes place first at the circumference of the center section at a critical rate of flow equal to $2/3$ of the fluidization rate in the absence of an obstacle.

If adhesion between particles of the solid phase is taken into consideration ($\sigma_s > 0$) the described above computation procedure is in the main retained, but must be supplemented by the determination of the length of the characteristic arc between the free surface and the point of transition to the fluidized state. The form of formula (1.8) implies that when the obstacle lies close to the free surface and the layer is reasonably deep, the fluidization process is linked with the formation of a discontinuity close to the bottom of the layer, and the obstacle has no effect.

5. To check the method of solving fluidization problems described above and in [1, 2] and, in particular, the results derived for the case of a cylindrical obstacle, special experiment was carried out on a plane model. The experiment consisted of measuring by highly sensitive sensors the variation of the layer porosity at various points close to the obstacle at several velocities of the fluidizing agent.

The experimental equipment consisted of a vertical column whose cross section had the form of a narrow rectangle 12.7×267 mm and the height was 1000 mm. A circular cylinder of 50 mm diameter was fixed in the middle between the column narrow faces at a distance of 450 mm above the gas flow distributor (a perforated plate). The cylinder axis was normal to the wide (front) faces of the column filled with small solid particles. The fluidizing agent was fed through the gas flow distributor. Particular attention was given to a thorough equalization of flow at the inlet.

For measuring local variations of the layer porosity in the neighborhood of the cylinder, the latter was surrounded by a set of eight plane capacitive probes, whose plates were located at the front faces of the column, as shown in the upper part of Fig. 3 (with retained relative dimensions). This arrangement made it possible to completely eliminate perturbations induced by probes.

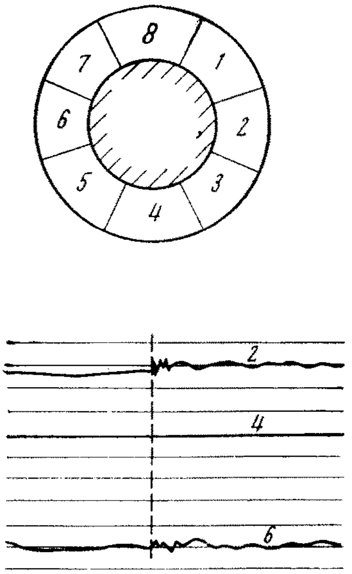

Fig. 3

Measurement of local variations of the layer porosity in the neighborhood of the cylinder by capacitive probes was apparently first described in [3].

The method used in our experiment is similar to that described, for instance, in [4]. It is distinguished by the inclusion in the probe circuit of a certain inductance for improving the sensitivity and reliability of the probes; the resulting oscillating circuit was tuned in resonance with a high-frequency generator. The variation of capacity produced a shift of natural oscillations of the probe circuit, and this was transformed into proportional voltage which was fed into the oscillograph circuit.

Experiments were carried out with quartz sand in a narrow range of 0.416 and 0.265 mm of average dimensions and of densities of 2.65 and 2.57 $g/cm^3$, respectively, fluidized by air.

To eliminate to the greatest possible extent the effect of adhesion between particles "reverse fluidization" was resorted to in our experiments, i.e. the high stream velocity at which the layer was fully fluidized was gradually reduced until the layer became completely stationary. In the course of this the layer capacitance in the cylinder neighborhood was continuously recorded by probes $1-8$ (Fig. 3). Typical oscillograms (curves are denoted by the probe number) are shown, as an example, in the lower part of Fig. 3 for stream velocities close to critical (the curves on the right of the vertical dash line relate to the stream velocity $U = 0.60\ U^\circ$, and on the left of it to $U = 0.58\ U^\circ$, where $U^\circ$ is the velocity at the beginning of fluidization in the absence of an obstacle).

It has been established that fluidization onset begins at points of the cylinder surface at its maximum cross section. The stream velocity at the beginning of fluidization was in all cases equal 55–60% of that in the absence of an obstacle, which within the accuracy of the experiment is completely in agreement with the results presented in Sect. 2.

We point out that the pattern at the beginning of fluidization obtained in Sect. 2 is in agreement with the results of observation of the behavior of the layer in the vicinity of a cylindrical obstacle under conditions of developed fluidization described in [5, 6].

In conclusion the authors express their gratitude to D. Harrison and S. Fahimi for their help in carrying out the experiment.

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# ON A METHOD OF REDUCING DUAL INTEGRAL EQUATIONS AND DUAL SERIES EQUATIONS TO INFINITE ALGEBRAIC SYSTEMS

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Mixed problems of the theory of elasticity, hydro- and aero-mechanics and of mathematical physics for the regions with partly infinite boundaries (a strip, layer, cylinder, wedge, cone, etc.), can often be reduced to studying dual integral equations. These equations usually involve the use of an integral transform generated by the Sturm-Liouville problem on a semi-infinite interval. Mixed problems for the finite regions (rectangle, circular plate, cylinder of finite length, sphere, etc.) can often be reduced to studying the dual series equations of some complete system of weight-orthonormed functions generated by the Sturm-Liouville problem on a finite interval. The present paper offers a method of reducing a wide class of such dual integral and series equations to infinite algebraic systems of special type. A way of investigating the infinite system obtained is indicated. The concept underlying the method was explained earlier by the author in [1].

**1.** Let a second order linear differential equation be given

$$(L - u^2)\, y = 0, \qquad Ly = r(x)\,[s(x)\, y']' + t(x)\, y \quad (a \leqslant x < \infty) \tag{1.1}$$

Here $s(x) > 0$ when $x \in (a, \infty)$ and $r(x)$ is a sign-definite function for $x \in (a, \infty)$. Let also the functions $y$ and $y'$ be bounded when $x \to \infty$ and

$$\alpha_2 y' + \alpha_1 y = 0 \tag{1.2}$$